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SOME QUESTIONS FROM THE USE OF REVERSIBLE FOLLOW-UP SYSTEMS WITH INDEPENDENT FOLLOW-UP AND FORCE REPRODUCTION CHANNELS

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ABSTRACT. Questions concerned with the use of independent follow-up and force reproduction channels in manipulators are discussed. The question of selecting the site of operator influence, as well as of the use of the slave mechanism as the reproduction channel output, are considered.

Let us take up a question of substantial practical significance, one that 73
arises with the realization, in a manipulator, of a system with independent follow-up and force reproduction channels [1], namely that of selecting the site of operator influence.

As we know [2, 3] there is no specific concept of system "input" and "output" for reversible follow-up systems. These definitions are purely arbitrary and any of the channels of a reversible drive can be taken as the master, and the other as the slave, as desired by the designer. Let us note that for symmetrical systems, which systems with combined [4] and intersecting channels may be, "input" and "output" are equivalent and, consequently, this question does not arise in their design.

Here we should refine the concept of system "symmetry." The technical literature on control systems for remote-controlled master-slave manipulators often gives different meanings for the concept of "symmetry." Some authors take drive symmetry to mean the symmetry of the drive structure [5], while others, in addition to symmetry of structure, also mean symmetry of drive parameters. According to the second definition, a symmetrical drive should provide equal "input" or "output" reactions to equal disturbing influences at the "output" or "input." In other words, the "output-input" and "input-output" transfer functions are identical for a symmetrical reversible follow-up system. This definition of symmetry, in our opinion, is more precise, and we shall adhere to it.

On the strength of the second definition of symmetry for a symmetrical reversible drive, the dynamics of drive operation do not depend on the site of operator and load action. Asymmetrical RFS (reversible follow-up systems), which include system with independent channels differ. Let us consider the

dynamics of its operation to include the effects of operator and load in order to determine the site of operator action in a system with independent channels. For simplicity in consideration of the question, let us take it that the moment sensors generating signals proportional to the moments acting at the outputs of the reproduction, M_1, and follow-up, M_2, channels have negligibly small deformations, and that they are installed directly on the manipulator's slave and master elements.

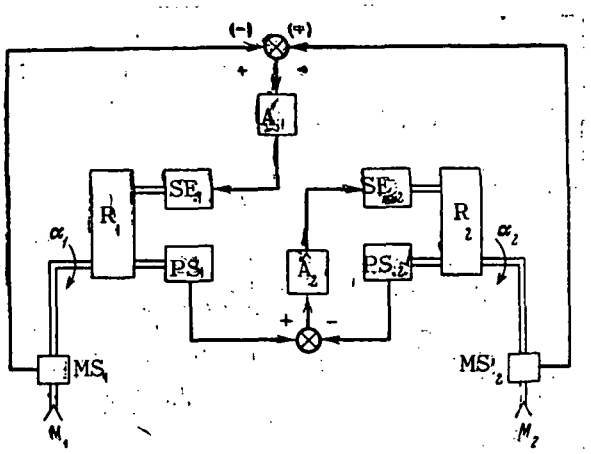

Figure 1.

Figure 1 is a block diagram of an RFS with independent follow-up and force reproduction channels, in which the following designations have been adopted: R_1, R_2 are reduction gears; SE_1, SE_2 are the slave elements in the reproduction and follow-up channels, respectively; PS_1, PS_2 are position sensors; MS_1 MS_2 are moment sensors; A_1, A_2 are amplifiers in the reproduction and follow-up channels, respectively; M_1, M_2 are the moments acting at the outputs of the reproduction and follow-up channels; α_1, α_2 are the generated angles for these channels. /74

System operation is as follows. The follow-up channel, PS_1, PS_2, A_2, SE_2, and R_2 generates position α_1. The reproduction channel, MS_1, MS_2, A_1, SE_1, and R_1 generates moment M_2. These two channels are connected such that the output of one is the input of the other, and vice versa.

Thus, when the operator, for example, acts from the reproduction channel side, M_1, the follow-up channel, controlled by position error $\delta_\alpha = (\alpha_1 - \alpha_2)$, will match the position of the shaft of the reproduction channel, α_1, and the reproduction channel, controlled by moment error $\delta_m = M_1 - M_2$, will help the operatior impart motion, generating at his output a moment coinciding in direction (sign) with the operator's moment.

Appearance of load moment M_2 at the follow-up channel will cause MS_2 to generate a signal which, depending on the direction of the sign of this moment,

will either help the operator impart motion or retard this motion, damping the effect on the channel for reproducing the signal M_1, from the moment sensor. The retarding action of moment M_2 will continue until operator moment M_1 becomes equal, or proportional, to load moment M_2; that is, until moment error δ_m disappears. System operation will be analogous when action by the operator is at the output of the follow-up channel.

The reproduction channel, generating a moment which coincides in direction (when there is no load moment) with the operator's moment at its output will impart motion in both cases. The signs of the signals in the comparison element in the reproduction channel indicated in Figure 1, generally speaking, are purely arbitrary and depend on the relationship and signs of the effective moments M_1 and M_2. The signs without parentheses are for the case when the operator acts at the reproduction channel output and the load has a retarding influence. The signs in parentheses are to indicate when the operator's influence at the follow-up channel output is the same as that of the load. It should be kept in mind that if the position sensors are installed ahead of the flexible couplings of the moment sensors, and if their deformations are taken into accoutn, the signs of the signals in the comparison element in the follow-up channel will also depend on the site of operator influence. /75

Based on the assumptions made above, the dynamics of the operation of RFS with independent channels, a block diagram of which is shown in Figure 1, can be described by the following system of differential equations

$$\left.\begin{aligned} n_1 &= W_1(p)\,\delta_m + W_{1m}(p)\,M_1; \\ n_2 &= W_2(p)\,\delta_n - W_{2m}(p)\,M_2; \\ \delta_m &= \tau_1 M_1 - \tau_2 M_2; \\ \delta_n &= n_1 - n_2. \end{aligned}\right\} \qquad (1)$$

Let us, for reasons of simplicity in considering the question, assume that

$$W_1(p) = \frac{n_1}{\delta_m} = \frac{k_1}{(T_{1m}p + 1)(T_{1a}p + 1)}; \qquad (2)$$

(3)

$$W_2(p) = \frac{n_2}{\delta_n} = \frac{k_2}{p\,(T_{2m}p + 1)(T_{2a}p + 1)}; \qquad (4)$$

$$W_{2m}(p) = \frac{n_2}{M_2} = \frac{k_{2m}}{(T_{2m}p + 1)}, \qquad (5)$$

where

n_1 is the speed at the output of the force reproduction channel;

n_2 is the speed at the output of the follow-up channel;

M_1 is the moment acting at the output of the reproduction channel;

M_2 is the moment acting at the output of the follow-up channel;

δ_m is the moment error acting at the input to the reproduction channel;

δ_n is the speed error acting at the input to the follow-up channel;

τ_1, τ_2 are the sensor conversion factors;

α_1 is the generated angle for the reproduction channel;

α_2 is the generated angle for the follow-up channel;

k_1 is the amplification factor for the reproduction channel circuit;

k_2 is the amplification factor for the follow-up channel circuit;

k_{1m}, k_{2m} are the coefficients of disturbing moments M_1 and M_2;

T_{1m}, T_{2m} are the time constants for the slave elements in the reproduction and follow-up channels including the moving parts in the mechanisms;

T_{1a}, T_{2a} are the electrical time constants for the amplifiers in the reproduction and follow channels;

$p = d/dt$ is the differential operator.

The solution of the system of equat $|Y| = -\frac{1 + W_2(p)}{W_{2m}(p)[W_1(p)\tau_1 + W_{1m}(p)]}$.

$$\left.\begin{aligned} M_1 &= \frac{\tau_2 W_1(p) W_2(p) + W_{2m}(p)}{W_{2m}(p)[W_1(p)\tau_1 + W_{1m}(p)]} n_1 - \frac{\tau_2 W_1(p)[1 + W_2(p)]}{W_{2m}(p)[W_1(p)\tau_1 + W_{1m}(p)]} n_2; \\ M_2 &= \frac{W_2(p)}{W_{2m}(p)} n_1 - \frac{1 + W_2(p)}{W_{2m}(p)} n_2. \end{aligned}\right\} \qquad (6)$$

The system of equations at (6) in matrix form is

(7)

By the analogy of the operation of an RFS to an electric four-terminal

network [8], [4], matrix [7] is the matrix of the conductivity of a reversible drive.

The solution of matrix equation (7) will be

$$[n] = [Y]^{-1}[M]. \tag{8}$$

The reciprocal matrix $[Y]^{-1}$ can be found from Cramer's rule

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$$[Y]^{-1} = \frac{1}{\Delta}\begin{bmatrix}\Delta_{11}\Delta_{21}\\ \Delta_{12}\Delta_{22}\end{bmatrix}; \qquad \Delta_{ik} = (-1)^{i+k} A_{ki}, \tag{9}$$

where $\Delta = |Y|$ is the determinant of matrix [Y]; A_{ki} is the minors of matrix [Y].

As Mason has shown [6, 7], when the determinant is a function of the minimum phase type (that is, has neither zeroes nor poles in the right half-plane) the system is stable, and the system is unstable when the case is reversed.

The determinant of a system with stable circuits is a stable function, so all that need be considered to have stability is the distribution of the zeroes of the determinant.

Proceeding from system of equations at (6), the determinant will be

$$|Y| = -\frac{1 + W_2(p)}{W_{2m}(p)[W_1(p)\tau_1 + W_{1m}(p)]}. \tag{10}$$

Or, when Eqs. (2), (3), (4), and (5) are considered Eq. (10) takes the form

$$|Y| = -\frac{(T_{1m}p + 1)[p(T_{2m}p + 1)(T_{2d}p + 1) + k_2]}{k_{2m}p[k_1\tau_1 + k_{1m}(T_{1d}p + 1)]}. \tag{11}$$

Hence, the characteristics equation of a reversible drive will be

$$F(p) = (T_{1m}p + 1)[k_2 + p(T_{2m}p + 1)(T_{2d}p + 1)] = 0. \tag{12}$$

This equation can be broken down into two parts, one of which depends on the parameters of the reproduction channel, the other on the parameters of the follow-up channel.

It is not difficult to see that system stability depends only on the parameters of the follow-up channel equation, the roots of which can be found by solving

$$F_1(p) = k_2 + p(T_{2m}p + 1)(T_{2a}p + 1) = 0. \quad (13)$$

The parameters of the force reproduction channel equation

$$F_2(p) = T_{1m}p + 1 = 0, \quad (14)$$

have no influence on system stability.

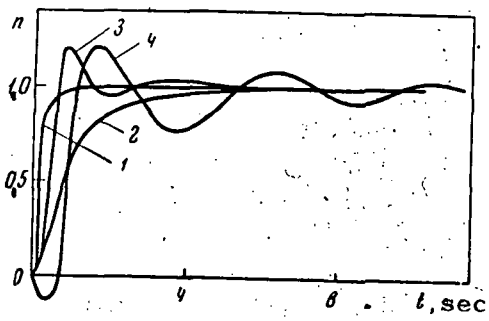

Figure 2.

The parameters of the characteristic equation, $F_1(p)$ and $F_2(p)$, for the system channel that will operate on the load, become variables that depend on the nature and magnitude of the load. If, for example, the load is purely inertial in nature, the magnitude of time constant, T_m, in Eqs. (13) or (14) will be a variable.

If the inertial load acts on the reproduction channel, stability of the system as a whole will not change, but its dynamic properties will deteriorate, with increase in load.

And if the inertial load acts on the follow-up channel, the dynamic properties of the system will change slightly, but the stability margin will be reduced and the system will become unstable when magnitude T_{2m} is equal to, or greater than, T_{mc}.

The value of T_{mc} for selected k_2 is determined by the expression /77

$$T_{mc} = (a + 1)/k_2, \quad (15)$$

where $a = T_m/T_a$.

Figure 2 illustrates the above hypothesis and shows the transient processes at the system output when there is a stepwise change in the disturbance at its input for the following values of the parameters: k_2 = 6.5 1/sec; k_1 = 10 rad/ /kgf-m·sec; $T_{1a}T_{2a}$ = 0.02 sec; $T_{1m} = T_{2m}$ = 0.1 to 0.8 sec; $k_{1m} = k_{2m}$ =

= 0.66 rad/kgf-m·sec; $\tau_1 = \tau_2 = 1$.

Curves 1 and 2 follow system reaction when the load acts on the reproduction channel; curves 3 and 4 follow system reaction when the load acts on the follow-up channel.

The transient processes are constructed from the expressions for the disturbance transfer functions, found using Eq. (3), and have the form

$$\Phi_{ik}(p) = n_i(p)/M_k(p) = \Delta_{ik}/\Delta. \tag{16}$$

Based on Eq. (16), and considering Eqs. (2-6), when the load is acting on the follow-up channel we will have

$$\Phi_{21}(p) = \frac{n_2}{M_1} = \frac{W_2(p)[W_1(p)\tau_1 + W_{1m}(p)]}{1 + W_2(p)} = \frac{k_2[k_1\tau_1 + k_{1m}(T_{1m}p + 1)]}{[p(T_{2m}p + 1)(T_{2a}p + 1) + k_2](T_{1m}p + 1)(T_{1a}p + 1)}. \tag{17}$$

When the load is acting on the reproduction channel, the expression for the transfer function will be

$$\Phi_{12}(p) = n_1/M_2 = \tau_2 W_1(p) = \tau_2 k_1/(T_{1a}p + 1)(T_{1m}p + 1). \tag{18}$$

Eqs. (17) and (18) will yield system sensitivity for the first and second cases, and is defined as the ratio of the steady-state speed to the steady-state moment.

$$S_2 = n_{2ss}/M_{1ss} = \tau_1 k_1 + k_{1m}; \tag{19}$$

$$S_{12} = n_{1ss}/M_{2ss} = \tau_2 k_1. \tag{20}$$

Since, in most cases, k_m is less than, or near, unity, as a practical matter it can be considered that when $\tau_1 = \tau_2$, system sensitivity will be the same when the effect is at the output of the follow-up channel, and at the output of the reproduction channel.

The moment transmission factor (scale factor) is determined from the moment transfer functions, which, using Eq. (8), will be

$$\Phi_{m1}(p) = \frac{M_1}{M_2} = \frac{\Delta_{12}}{\Delta_{11}} = \frac{Y_{12}(p)}{Y_{22}(p)} = \frac{\tau_2 W_1(p)}{\tau_1 W_1(p) + W_{1m}(p)}; \tag{21}$$

$|M'| =$
$|M''| =$
$|M'''| =$

$$\Phi_{m2}(p) = \frac{M_2}{M_1} = \frac{\Delta_{21}}{\Delta_{22}} = \frac{Y_{21}(p)}{Y_{11}(p)} = \frac{W_2(p)\,[W_1(p)\,\tau_1 + W_{1m}(p)]}{\tau_2 W_1(p)\,W_2(p) + W_{2m}(p)}. \qquad (22)$$

Substituting the values of the polynomials $W_1(p)$, $W_2(p)$, $W_{1m}(p)$, and $W_{2m}(p)$ from Eqs. (2-5) into Eqs. (22) and (23), we obtain

$$\Phi_{1m}(p) = \frac{k_1\tau_2}{\tau_1 k_1 + k_{1m}(T_{1a}p + 1)}; \qquad (23)$$

$$\Phi_{m1}(0) = \frac{M_{1ss}}{M_{2ss}} = m_1 = \frac{\tau_2 k_1}{\tau_1 k_1 + k_{1m}}; \qquad (24)$$

$$\Phi_{m2}(p) = \frac{k_2\,[k_1\tau_1 + k_{1m}(T_{1a}p + 1)]}{\tau_2 k_1 k_2 + p\,(T_{\mu m}p + 1)\,(T_{1a}p + 1)\,(T_{2a}p + 1)\,k_{2m}}; \qquad (25)$$

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$$\Phi_{m2}(0) = \frac{\tau_1 k_1 + k_{1m}}{\tau_2 k_1} = m_2 = \frac{M_{2ss}}{M_{1ss}}. \qquad (26)$$

Thus, what follows from Eqs. (24) and (26) is that the moment perceived by the operator as attributable to the reproduction channel will be somewhat less (when $\tau_1 = \tau_2$) than the load moment ($m_1 < 1$), but if the operator is acting at the input to the follow-up channel, the moment he perceives will be somewhat greater than the actual magnitude (when $\tau_1 = \tau_2$ $m_2 > 1$).

The scale factors for the values of the parameters assumed above, will be, respectively

$$\tau_1 = \tau_2; \quad m_1 = \frac{10}{10 + 0{,}66} = 0{,}94; \quad m_2 = \frac{10 + 0{,}66}{10} = 1{,}066.$$

The influence of the magnitude of factor k_m will be negligible as the reproduction channel transmission factor, k_1, increases. Here the transmission scale factor will depend completely on the ratio of the moment sensor conversion factors

$$m_1 = \tau_2/\tau_1 = 1/m_2. \qquad (27)$$

An analysis of the influence of the load on system operation in terms of where it is acting and including the elasticity of the kinematic transmissions in the slave mechanisms in the follow-up and reproduction channels is quite difficult to make because of the increase in the degree of the characteristic

$$\left.\begin{aligned} |M'| &= |Y'|\,|n'|; \\ |M''| &= |Y''|\,|n''|; \\ |M'''| &= |Y'''|\,|n'''|. \end{aligned}\right\}$$

equation. However, we shall try to establish certain theses for this case.

If, for example, the elasticity of the kinematic transmission in the reproduction channel is considered, the following system of differential equations will apply in this case

$$\left.\begin{aligned} M_0 &= \left[A_0(p) + \frac{c_1}{p}\right] n_0 - \frac{c_1}{p} n_1; \\ M_1 &= \frac{c_1}{p} n_0 - \frac{c_1}{p} n_1; \\ M_1 &= \frac{\tau_2 W_1(p) W_2(p) + W_{2m}(p)}{W_{2m}(p)[\tau_1 W_1(p) + W_{1m}(p)]} n_1 - \frac{\tau_2 W_1(p)[1 + W_2(p)]}{W_{2m}(p)[\tau_1 W_1(p) + W_{1m}(p)]} n_2, \\ M_2 &= \frac{W_2(p)}{W_{2m}(p)} n_1 - \frac{1 + W_2(p)}{W_{2m}(p)} n_2; \\ A_0 &= J_0 p + b_0, \end{aligned}\right\} \quad (28)$$

where

c_1 is the stiffness factor for the kinematic transmission in the slave mechanism of the reproduction channel;

J_0 is the moment of inertia of the moving parts ahead of the flexible link;

b_0 is the coefficient of velocity in the flexible link.

The system of equations at (28) is in matrix form

$$\left.\begin{aligned} |M'| &= |Y'||n'|; \\ |M''| &= |Y''||n''|, \end{aligned}\right\} \quad (29)$$

where

$$|M'| = \left|\frac{M_0}{M_1}\right|; \quad |M''| = \left|\frac{M_1}{M_2}\right| -$$

are the matrices of the moments of the first and second systems,

$$|n'| = \left|\frac{n_0}{n_1}\right|; \quad |n''| = \left|\frac{n_1}{n_2}\right| -$$

are the velocities matrices, and

$$|Y'| = \begin{vmatrix} Y'_{11}(p) & Y'_{12}(p) \\ Y'_{21}(p) & Y'_{22}(p) \end{vmatrix}; \quad |Y''| = \begin{vmatrix} Y''_{11}(p) & Y''_{12}(p) \\ Y''_{21}(p) & Y_{22}(p) \end{vmatrix} -$$

are the conductivity matrices.

The parameters of a generalized system made up of a chain coupling of two

$$\left.\begin{aligned} |M'| &= |Y'||n'|; \\ |M''| &= |Y''||n''|; \\ |M'''| &= |Y'''||n'''|. \end{aligned}\right\}$$

reversible systems (by analogy with four-terminal networks) are determined by the following relationships:

$$Y_{11}(p) = \frac{-|y'| + Y'_{11}Y''_{11}}{-Y'_{22} + Y''_{11}} = \Delta_{22}; \tag{30}$$

$$Y_{12}(p) = -\frac{Y'_{12}Y''_{12}}{-Y'_{22} + Y''_{11}} = -\Delta_{12}; \tag{31}$$

$$Y_{21}(p) = \frac{Y''_{21}Y'_{21}}{-Y'_{22} + Y''_{11}} = -\Delta_{21}; \tag{32}$$

$$Y_{22}(p) = \frac{Y'_{22}Y''_{22} - |y''|}{-Y'_{22} + Y''_{11}} = \Delta_{11}; \tag{33}$$

$$|y| = \Delta = -\frac{Y''_{22}|y'| - Y'_{11}|y''|}{-Y'_{22} + Y''_{11}}, \tag{34}$$

where

$|y'|$; $|y''|$ are the determinants of the composite systems;

$\Delta = (y)$ is the determinant of the generalized system;

Δ_{ik} is the minors of the generalized system.

As before, the characteristic equation will be

$$F(p) = \Delta = |y| = 0.$$

Using Eq. (34), and considering the system of equations at (28), we have, for the characteristic equation,

$$F(p) = -\frac{[1 + W_2(p)]\{A_0(p)[c_1\tau_1 W_1(p) -$$
$$W_{2m}(p)[p + c_1\tau_1 W_1 (p)$$

(35)

As may be seen from Eq. (35), the numerator can have two factors, so system stability is determined by the roots of each, and when Eqs. (2-5) are included, we have

$$F_1(p) = [p(T_{2m}p + 1)(T_{2a}p + 1) + k_2] = 0; \tag{36}$$

$$F_2(p) = (T_{1m}p + 1)(T_{1a}p + 1)(J_0p^2 + b_0p + c_1) +$$
$$+ c_1(J_0p + b_0)[\tau_1 k_1 + k_{1m}(T_{1a}p + 1)] = 0. \tag{37}$$

$$\left.\begin{aligned} |M'| &= |Y'||n'|; \\ |M''| &= |Y''||n''|; \\ |M'''| &= |Y'''||n'''|. \end{aligned}\right\}$$

If we disregard T_{1a}, we can, by opening the brackets and grouping the terms in Eq. (37), obtain

$$F_2(p) = T_{1m}J_0p^3 + (J_0 + b_0T_{1m})\,p^2 + [b_0 + c_1T_{1m} + c_1(\tau_1k_1 + k_{1m})\,J_0]\,p + \\ + c_1[1 + b_0(\tau_1k_1 + k_{1m})] = 0. \tag{38}$$

Proceeding from Eq. (38), the condition for stability will be

$$c_1(k_1\tau_1 + k_{1m})\,J_0^2 + b_0T_{1m}(b_0 + c_1T_{1m}) > 0, \tag{39}$$

which always is feasible for any parameter values.

Thus, a changing inertial load on the follow-up channel will change T_{2m}, and, as shown above, system stability will deteriorate. Loading the reproduction channel will change J_0, but this will not cause a deterioration in stability conditions. /80

If we consider the elasticity of the kinematic transfers of the reproduction channel as well as of the follow-up channel, then equations of the elastic transfer of the follow-up channel will also be added to system of equations (29)

$$\left.\begin{aligned} M_2 &= \frac{c}{p}(n_2 - n_{fu}); \\ M_1 &= \frac{c_2}{p}n_2 - \left[A_s(p) + \frac{c_2}{p}\right]n_{fu} \\ A_s(p) &= J_sp + b_s \end{aligned}\right\} \tag{40}$$

where

n_{fu} is the speed at the output of the follow-up channel, including the flexible transmission;

M_1 is the moment at the output;

J_s is the moment of inertia of the moving parts in front of the flexible element (read from the output);

b_s is the speed losses in the flexible link.

A reversible system as a whole can be represented in the form of a chain connection of three composite reversible systems with matrix equations

(41)

$n'/;$
$n''/;$
$n'''/.$ } 108

Here the following equations can be used to find the system's generalized parameters

$$Y_{11}(p)=\frac{Y'_{11}Y''_{11}Y'''_{11}+|y'|(Y''_{22}-Y'''_{11})-Y'_{11}|y''|}{Y'_{22}Y''_{22}-|y''|+Y'''_{11}(Y''_{11}-Y'_{22})}=\Delta_{22}; \tag{42}$$

$$Y_{12}(p)=\frac{Y'_{12}Y''_{12}Y'''_{12}}{Y'_{22}Y''_{22}-|y''|+Y'''_{11}(Y''_{11}-Y'_{22})}=-\Delta_{12}; \tag{43}$$

$$Y_{22}(p)=\frac{-Y'_{22}Y''_{22}Y'''_{22}+Y'''_{22}|y''|-|Y'''|(y'_{11}-Y'_{22})}{Y'_{22}Y''_{22}-|y''|+Y'''_{11}(Y''_{11}-Y'_{22})}=\Delta_{11}; \tag{44}$$

$$Y_{21}(p)=\frac{Y'_{21}Y''_{21}Y'''_{21}}{Y'_{22}Y''_{22}-|y'''|+Y'''_{11}(Y''_{11}-Y'_{22})}=-\Delta_{21}; \tag{45}$$

$$|y|=-\frac{-Y''_{22}Y'''_{22}|Y'|+Y'_{11}y'''_{22}|y''|+|y'||y'''|-Y'_{11}Y''_{11}|y'''|}{Y'_{22}Y''_{22}-|y''|+Y'''_{11}(Y''_{11}-Y'_{22})}=\Delta; \tag{46}$$

where

$\left|y'\right|$; $\left|y''\right|$; $\left|y'''\right|$ are the determinants of the composite systems;

$\Delta = \left|y\right|$ is the determinant of the generalized system;

Δ_{ik} is the minors of the generalized system.

Substituting the conductivity values from Eq. (28) and Eq. (40) into Eqs. (42) - (46) for the system determinant, we obtain

$$|y|=-\frac{\{A_S(p)[p(1+W_2(p))+c_2W_{2m}(p)]+c_2(1+W_2(p))\}\{A_0(p)[p+c_1(\tau_1W_1(p)+W_{1m}(p)]+c_1\}+c_2A_S(p)\tau_2W_1(p)W_2(p)[pA_0(p)+c_1]}{\{p+c_1[\tau_1W_1(p)+W_{1m}(p)]\}\{c_2W_{2m}(p)+p[1+W_2(p)]\}+pc_2\tau_2W_1(p)W_2(p)} \tag{47}$$

For disturbance transfer functions

$$\Phi_{21}(p)=\frac{n_S}{M_0}=\frac{\Delta_{21}}{\Delta}=\frac{c_1c_2W_2(p)[\tau_1W_1(p)+W_{1m}(p)]}{(A_S(p)\{p[1+W_2(p)]+c_2W_{2m}(p)\}+c_2[1+W_2(p)])A_0(p)\times \{p+c_1[\tau_1W_1(p)+W_{1m}(p)]+c_1\}+c_2A_S(p)\tau_2W_1(p)W_2(p)\times [pA_0(p)+c_1];} \tag{48}$$

$$\Phi_{12}(p)=\frac{n_0}{M_1}=\frac{\Delta_{12}}{\Delta}=\frac{c_1c_2\tau_2W_1(p)[1+W_2(p)]}{(A_S(p)\{p(1+W_2(p)]+c_2W_{2m}(p)\}+c_2[1+W_2(p)])(A_0(p)\times \{p+c_1[\tau_1W_1(p)+W_{1m}(p)]+c_1\}+c_2A_S(p)\tau_2W_1(p)W_2(p)\times [pA_0(p)+c_1].} \tag{49}$$

/81

For moment transfer functions

$$\Phi_{m1}(p) = \frac{M_0}{M} = \frac{\Delta_{12}}{\Delta_{11}} = \frac{c_1 c_2 \tau_2 W_1(p)[1 + W_2(p)]}{\{A_s(p)[p(1 + W_2(p)) + c_2 W_{2m}(p)] + c_2(1 + W_2(p)\} \times} \times \\ \times [p + c_1(\tau_1 W_1(p) + W_{1m}(p)] + c_2 p A_s \tau_2 W_1 W_2 ; \quad (50)$$

(51)

For the sensitivity and scale factor for transmission of moments, determined for steady-state conditions from Eqs. (48) - (51) and including Eqs. (2) - (5), we will have

$$S_{21} = \frac{n_{s\,ss}}{M_{0\,ss}} = \frac{k_1\tau_1 + k_{1m}}{b_0(\tau_1 k_1 + k_{1m}) + b_c \tau_2 k_1 + 1} ; \quad (52)$$

(53)

$$m_1 = \frac{M_{0\,ss}}{M_{1\,ss}} = \frac{\tau_2 k_1 k_2}{(\tau_1 k_1 + k_{1m})(b_c k_{2m} + k_2)} ; \quad (54)$$

$$m_2 = \frac{M_{1\,ss}}{M_{0\,ss}} = \frac{k_2(\tau_1 k_1 + k_{1m})}{k_{2m}[b_0(\tau_1 k_1 + k_{1m}) + 1]\tau_2 k_1 k_2} . \quad (55)$$

If we ignore the terms containing the parameters $b_o \ll 1$ and $b_s \ll 1$ in Eqs. (52) - (55), the relationships obtained will be identical with Eqs. (19), (20), (24), and (26) established for a reversible system without consideration for flexible links.

The characteristic equation for the generalized system will be

$$F(p) = |y| = \{A_s(p)[p(1 + W_2(p)) c_2 W_{2m}(p)] + \\ + c_2(1 + W_2(p)\}\{A_0(p)[p + c_1(\tau_1 W_1(p) + W_{1m}(p))] + c_1\} + \\ + c_2 A_s(p)\tau_2 W_1(p) W_2(p)[pA_0(p) + c_1] = 0. \quad (56)$$

The parameters of the characteristic equation depend on the load. If the load is at the output of the follow-up channel, the variables will be the parameters of the polynomial $A_s(p) = J_s p + b_s$. If the load is inertial, the moment of inertia, J_s, will change from $J'_s \ll J_s \ll J'_s + J_1$ where J'_s is the

value of the moment of inertia for system idling speed and J_1 is the load's moment of inertia. The variables will be the parameters of the polynomial $A_0(p) = J_0 p + b_0$ if the inertial load is at the output of the reproduction channel. The moment of inertia, J_0, as in the preceding case, will vary in limits $J_0' \ll J_0 \ll J_0' + J_1$.

Characteristic Eq. (56) will not factor in two, so analysis of the influence of the load on system operation in terms of where applied requires cumbersome mathematical computations, and is not presented here.

We note that in a system with flexible kinematic links, the influence of a load at the output of the reproduction channel, as in the preceding case, has much less effect on operational stability than when the influence is at the output of the follow-up channel. /82

System sensitivity does not, practically speaking, depend on where load and operator act, and deterioration in the dynamic properties of the system with increase in load is fully permissible for manipulators (because of limited operator capabilities and safe working conditions), so the use of the manipulator slave mechanism as the output of the force reproduction channel, and of the master mechanism as the output of the follow-up channel, can be considered more rational.

Figure 3 is the block diagram of a reversible follow-up system with independent follow-up and force reproduction channels. This scheme derives from Eqs. (2) - (5), (29), and (41) and includes a load acting the reproduction channel output.

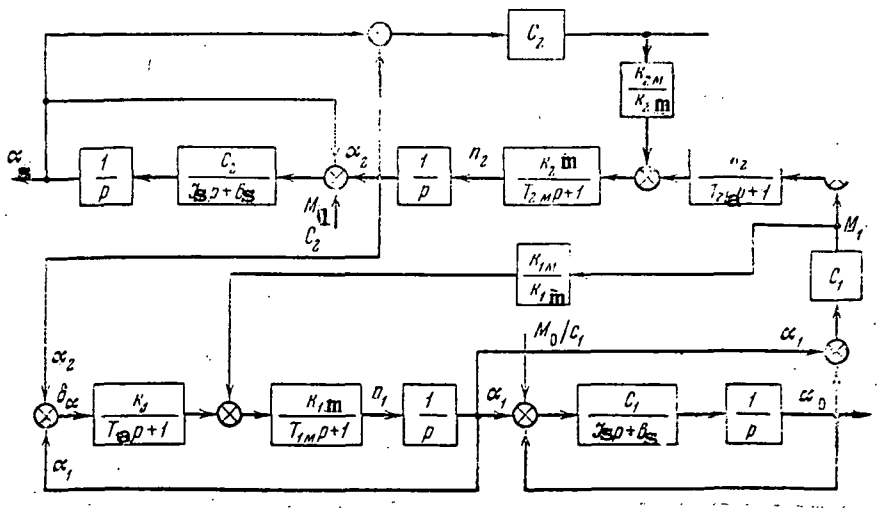

Figure 3.

Thus, in remotely-controlled master-slave manipulators based on reversible follow-up systems with independent follow-up and force reproduction channels, it is desirable to use the manipulator's slave mechanism as the outputs of the reproduction channels of reversible systems.

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